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Phase Transitions in the Embryo Universe [and Discussion]

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Phase transitions in the embryo Universe

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Grand unified theories suggest that one or more phase transitions should occur when $kT \approx 10^{14}$ GeV ($T \approx 10^{27}$ K), which in standard cosmology would have occurred at $t \approx 10^{-35}$ s. If such a transition happens quickly, then an unacceptably large number of magnetic monopoles would be produced. A slow transition would lead to an inflationary Universe, providing a possible solution to the horizon and flatness problems. However, the inflationary scenario requires a mechanism to trigger the phase transition. A suggestion made independently by Linde and by Albrecht & Steinhardt appears promising, but the details have not yet been worked out.

In this paper I shall discuss the phase transitions of grand unified theories (GUTs), and the role that they may have played in the very early evolution of our Universe. In particular, I shall discuss a scenario that I call the inflationary Universe (Guth 1981). In this scenario the Universe supercools by many orders of magnitude below the critical temperature of a GUT phase transition, and in the process it expands by many orders of magnitude. If this scenario is valid it would mean that GUT mechanisms are responsible not only for the production of all the baryons in the Universe, but for the production of all the matter, energy and entropy as well.

The inflationary scenario is allowed by GUTs – it is even quite plausible – but it is not required by GUTs. Whether inflation occurs or not depends sensitively on undetermined parameters in the Higgs field potential. The scenario is, however, very attractive because it can possibly solve three very important cosmological problems: the monopole problem, the horizon problem, and the flatness problem. I shall explain the meaning of these problems below.

I wish to emphasize at the outset that the inflationary scenario is incomplete at the present time. It is not yet clear if the period of rapid expansion can be smoothly terminated to give rise to the present, non-inflationary Universe. However, there has been a promising development in this direction in just the past few months. Linde (1982) and Albrecht & Steinhardt (1982) have independently proposed a modified ending to the inflationary scenario, in which the observed Universe emerges from a single bubble or fluctuation associated with the phase transition. I shall describe their suggestion in more detail below, but the complete details have not yet been worked out.

Let me begin by describing the nature of the GUT phase transition. For concreteness I shall discuss the minimal SU(5) model of Georgi & Glashow (1974); the main points, however, will be fairly model-independent.

In the SU(5) model, the full gauge symmetry is broken to the subgroup SU(3) × SU(2) × U(1) by a set of Higgs fields Φ, which transform according to the adjoint representation of SU(5). That is, Φ represents a traceless, Hermitian 5×5 matrix of fields, with 24 independent components.

The symmetry breaking is accomplished by the fields Φ acquiring an expectation value of the form

$$\langle\Phi\rangle = \phi \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 & -\frac{3}{2} \end{bmatrix}, \tag{1}$$

or a form gauge-equivalent to this. The phase transition occurs because this non-zero expectation value is destroyed by thermal fluctuations at high temperatures (Kirzhnits & Linde 1972; Weinberg 1974; Dolan & Jackiw 1974), just as thermal fluctuations destroy the expectation value of a magnetic spin of a ferromagnet at temperatures above the Curie point. Thus, at the highest temperatures,

$$\langle\Phi\rangle = 0. \tag{2}$$

For a wide range of parameters, there is an intermediate phase (Guth & Tye 1980) in which

$$\langle\Phi\rangle = \phi' \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -4 \end{bmatrix}. \tag{3}$$

The critical temperature T_c for these phase transitions is typically of order 10^{14} GeV.

I am using units in which $\hbar = c = k = 1$, and I shall use the gigaelectronvolt as my fundamental unit. The approximate conversions are given by

$$\begin{aligned} 1\,\text{GeV} &\approx 10^{13}\,\text{K} \approx 10^{-24}\,\text{g}; \\ 1\,\text{GeV}^{-1} &\approx 10^{-14}\,\text{cm} \approx 10^{-24}\,\text{s}. \end{aligned} \tag{4}$$

Before going on to discuss cosmology, let me point out a defect in this description: the expectation value of Φ is a gauge-dependent quantity, and it is not clear that the value that it takes when computed with any particular gauge condition has any real physical meaning. It is clear that these phase transitions would take place if the gauge coupling constant were set equal to zero. With a weak gauge coupling (as in GUTs), it is conceivable that the discontinuities that one expects in a phase transition are in fact 'rounded' by the gauge interactions.

Now let me discuss what may be called the standard model of the very early Universe. The Universe is described by a Robertson–Walker metric. Whether it is open, closed or flat, the curvature is negligible at very early times. Thus

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + R^2(t)\,\mathrm{d}\boldsymbol{x}^2. \tag{5}$$

The universe expands adiabatically, with a mass density ρ dominated by the radiation of effectively mass less (i.e. $M \ll T$) particles at temperature T:

$$\rho = cT^4. \tag{6}$$

For the minimal SU(5) theory, $c \approx 50$ at the highest temperatures. As long as the number of effectively massless particle species does not change, the temperature varies with time t according to

$$T^2 = M_P/2\gamma t, \tag{7}$$

where $\gamma = (\frac{8}{3}\pi c)^{\frac{1}{2}} \approx 20$, and $M_P = G^{-\frac{1}{2}} = 1.2 \times 10^{19}$ GeV is the Planck mass. One has RT = constant, so $R \propto t^{\frac{1}{2}}$. When $T = 10^{14}$ GeV, the time is 10^{-35} s, the energy density is 10^{75} g cm^{-3}, and the radius of the region that will evolve to become our observed Universe is only about 10 cm. In the standard model, the one or more phase transitions occur without any significant supercooling.

As I mentioned in my introduction, the standard model has three important problems, which I shall now discuss. The first is the monopole problem. Any GUT in which a simple group is broken eventually to $SU(3) \times U(1)$ (i.e. quantum chromodynamics and electromagnetism) will necessarily (Goddard & Olive 1978) contain magnetic monopoles of the 't-Hooft–Polyakov ('t-Hooft 1974; Polyakov 1974) type. These monopoles are topologically stable knots that can be tied in the Higgs field expectation value. Now consider the behaviour of the Higgs field during the phase transition. For $T < T_c$, it is thermodynamically favourable for the Higgs field to align uniformly over large distances. However, it takes time for these correlations to be established. Causality implies that the Higgs field correlation length ξ must be less than the horizon length $2t$, which is the distance that a light pulse could have travelled since the initial singularity. Since the monopoles are knots in the Higgs expectation value, their density is given roughly (Kibble 1976) by

$$n_M \approx 1/\xi^3 \geqslant 1/8t^3. \tag{8}$$

This implies that the ratio of the monopole density to the entropy density just after the phase transition was bounded by

$$n_M/s \gtrsim 10^{-13}. \tag{9}$$

This ratio is unchanged by the expansion of the Universe, and Preskill (1979) has shown that monopole–antimonopole annihilation is totally ineffective at these densities. (See also Zel'dovich & Khlopov (1978).) Entropy is essentially conserved in this model, so the ratio n_M/s should be about the same today. However, the monopoles are extraordinarily heavy, with $M_M \approx 10^{16}$ GeV. If such a large density of monopoles were present, their mass density would be 10^{12} times the critical mass density of our Universe. Since that is impossible, we must understand how monopole production was suppressed. That is the monopole problem. (Possible solutions to the monopole problem has been proposed by Langacker & Pi (1980) and by Dicus & Teplitz (1981).)

The second problem is the horizon problem, first pointed out by Rindler (1956). The observational basis for this problem is the uniformity of the cosmic background radiation, which is known to be isotropic to about one part in 10^3. This fact is particularly difficult to understand when one considers the existence of the horizon length, the maximum distance that light could have travelled since the beginning of time. Consider two microwave antennas pointed in opposite directions. Each is receiving radiation that is believed to have been emitted (or 'decoupled') at the time of hydrogen recombination, at $t \approx 10^5$ years. At the time of emission, these two sources were separated from each other by over 90 horizon lengths (Guth 1982). The problem is to understand how two regions over 90 horizon lengths apart came to be at the same temperature at the same time. Within the standard model this large-scale homogeneity is simply assumed as an initial condition.

The third problem is the flatness problem, which was first pointed out by Dicke & Peebles (1979). For any given value of the Hubble 'constant' H, there is a critical mass density

$$\rho_c = 3H^2/8\pi G, \tag{10}$$

which gives a precisely flat ($k = 0$) Universe. Today one can state conservatively that $\rho/\rho_c \equiv \Omega$ lies in the range

$$0.01 < \Omega < 10. \tag{11}$$

No one is surprised by how narrowly this range brackets $\Omega = 1$. However, within the evolution of the standard model, the value $\Omega = 1$ is an *unstable* equilibrium point. Thus, to be near $\Omega = 1$ today, the universe must have been very near to $\Omega = 1$ in the past. When T was 1 MeV, Ω had to equal unity to an accuracy of one part in 10^{15}. When T was 10^{14} GeV, Ω had to be equal to unity to within one part in 10^{49}. In the standard model this precise fine-tuning is simply put in as an initial condition. However, I feel that the model is believable only if a mechanism to create this fine-tuning can be found.

The horizon and flatness problems are peculiarities of the initial conditions of the standard model. It is therefore conceivable that they are solved at the level of quantum gravity. However, I shall show that a plausible solution already exists at the level of GUT phase transitions, a topic much more within our reach. Even if the solution really occurs at the level of quantum gravity, it is conceivable that the mechanism closely resembles the inflationary scenario.

Now I am ready to explain the inflationary Universe, using the new ending suggested by Linde (1982) and Albrecht & Steinhardt (1982), and I shall try to point out which aspects of the scenario have not yet been properly studied.

The new ending requires that the parameters of the Higgs potential be adjusted to obey the Coleman–Weinberg (1973) condition, $\partial^2 V/\partial\phi^2 = 0$ at $\phi = 0$. Including the one-loop quantum corrections, the SU(5) potential takes the form

$$V(\phi) = \tfrac{5625}{1024}\,(g^4/\pi^2)\,[\phi^4 \ln(\phi^2/\sigma^2) + \tfrac{1}{2}(\sigma^4 - \phi^4)], \tag{12}$$

where ϕ is defined by (1), $g^2/4\pi \approx \frac{1}{45}$ is the gauge coupling, and $\sigma \approx 4.5 \times 10^{14}$ GeV. The form of this potential is shown in figure 1. The minimum lies at $\phi = \sigma$, corresponding to the true vacuum. The point $\phi = 0$ is an equilibrium point that is just barely unstable at $T = 0$ but is stabilized by finite temperature corrections. I shall refer to the field configuration $\phi = 0$ as the false vacuum (although this term is normally reserved for configurations that are classically stable at zero temperature).

The starting point of a cosmological scenario is somewhat a matter of taste and philosophical prejudice. Some physicists find it plausible that the Universe began in some highly symmetrical state, such as a De Sitter space. I prefer to believe that the Universe began in a highly chaotic state; one advantage of the inflationary scenario, from my point of view, is that it appears to allow a wide variety of starting configurations. I require only that the initial universe is hot ($T > 10^{14}$ GeV) in at least some places, and that at least some of these regions are expanding rapidly enough so that they will cool to T_c before gravitational effects reverse the expansion.

In these hot regions, thermal equilibrium would imply that $\langle\Phi\rangle = 0$. Such a region will cool to T_c, and it will then supercool below T_c. The energy density ρ will approach $\rho_0 \equiv V(\phi = 0)$. (Actually, though, the Universe has not had time at this point to thermalize. Thus I need to assume that there are some regions of high energy density with $\langle\Phi\rangle \approx 0$, and that some of these regions lose energy with Φ being trapped in the false vacuum.) Assuming that this local region is sufficiently homogeneous and isotropic to be approximated by a Robertson–Walker metric, then the constant energy density leads to exponential expansion,

$$R(t) \propto e^{\chi t}, \tag{13}$$

where

$$\chi = (\tfrac{8}{3}\pi G\rho_0)^{\frac{1}{2}} \approx 10^{10}\,\text{GeV}. \tag{14}$$

The state of pure exponential expansion is a De Sitter space. As long as $\rho = \rho_0$, it is very stable against any small perturbation: initial inhomogeneities or anisotropies are damped on the

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timescale of χ^{-1}. (Advocates of an initial De Sitter space are welcome to join the scenario at this point.)

As the space continues to supercool and expand, the energy density is fixed at ρ_0. Thus the total matter energy is increasing! This seems to violate our naïve notions of energy conservation, but we must remember that the gravitational field can exchange energy with the matter fields. The energy–momentum tensor for matter obeys a convariant conservation equation, which in the Robertson–Walker metric reduces to

$$\frac{\mathrm{d}}{\mathrm{d}t}(R^3\rho) = -p\frac{\mathrm{d}}{\mathrm{d}t}(R^3), \tag{15}$$

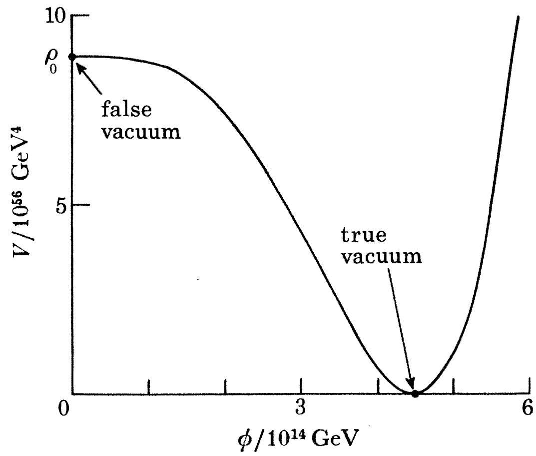

FIGURE 1. The Coleman–Weinberg potential for the SU(5) adjoint Higgs field.

where p is the pressure. In the limit that the temperature goes to zero, the false vacuum state is completely Lorentz-invariant. The energy–momentum tensor must then have the form

$$T_{\mu\nu} = \rho_0 g_{\mu\nu}, \tag{16}$$

from which it follows that

$$p = -\rho_0. \tag{17}$$

The false vacuum has a large and negative pressure! Equation (15) is then satisfied identically, with the energy of the expanding gas increasing owing to the negative pressure. If the space were asymptotically Minkowskian it would be possible to define a conserved total energy (matter plus gravitational). However, in the Robertson–Walker metric, no such conservation law can be defined.

Let me digress a moment to discuss the contents of the observed Universe. I have just explained that matter energy is not conserved, but can be increased dramatically by exponential expansion. If baryon number is also non-conserved (as in GUTs), then the Universe is, as far as we know, devoid of any conserved quantities (Atkatz & Pagels 1981). In that case, it is very tempting to believe that the Universe began from nothing, or from almost nothing. The inflationary model illustrates the latter possibility.

Now let me return to my region of space that is supercooling into a De Sitter phase. It is a striking fact about De Sitter space that quantum fluctuations mimic thermal effects at a Hawking temperature (Gibbons & Hawking 1977)

$$T_{\mathrm{H}} = \chi/2\pi. \tag{18}$$

In our case, $T_H \approx 10^9$ GeV. Calculations on the decay of the supercooling phase have been carried out only in Minkowski space, omitting the effects of gravitation (including the Hawking temperature). These calculations (Sher 1981; Albrecht & Steinhardt 1982) indicate that the supercooling continues until $T \approx 10^{7-8}$ GeV, at which point the calculations break down. Since gravitational effects are presumably quite important below the Hawking temperature, this part of the scenario is not yet well understood. (A first step toward the inclusion of the gravitational effects has been taken by Vilenkin (1982).)

The crucial assumption is that within the supercooling region, small regions of fluctuation will form, owing to either thermal or quantum processes. Linde refers to these small regions as bubbles, but Albrecht & Steinhardt point out that highly non-spherical fluctuations are also quite likely. I shall call them fluctuation regions (FRs). The important assumption (justified in Minkowski space calculations) is that the initial magnitude of the fluctuation in ϕ is of order 10^8 GeV or less. The field will then obey the classical equations of motion; it will slide down the potential, but on a timescale that is slow compared with the exponential expansion rate. In this way a single FR can grow to be larger than (or much larger than) the observed Universe, which is then assumed to lie within one such region.

When the ϕ field reaches the steep part of the potential, it falls quickly to the bottom and oscillates about the minimum. The timescale of this motion is a typical GUT time of $(10^{14}\,\text{GeV})^{-1}$, which is very fast compared with the expansion rate. The Higgs field oscillations are then quickly damped by the couplings to the other fields, and the energy is rapidly thermalized. (The Higgs field oscillations correspond to a coherent state of Higgs particles; the damping is simply the decay into other species.) The release of this energy (which is just the latent heat of the phase transition) raises the temperature to the order of 10^{14} GeV. From here onwards the standard scenario ensues, including the production of a net baryon number.

Before supercooling, the region of space that evolves to become the observed Universe may very well have contained no particles at all. Thus all the matter, energy and entropy of the observed Universe was generated by the false vacuum. It is the energy from the false vacuum that evolves to become the galaxies, stars, planets and human beings.

(The above scenario is in contrast to my original proposal, in which I did not realize the possibility of the slow classical motion of the Higgs field. I therefore tried to build the Universe from the collisions of many bubbles, but I never found a mechanism that could do this without introducing gross inhomogeneities from the randomness of the bubble formation process (Guth 1981; Guth & Weinberg 1982).)

I must now explain how the monopole, horizon and flatness problems disappear in this scenario.

The monopole problem was caused by the smallness of the Higgs correlation length in the standard model. In this scenario, the entire observed Universe evolves from a single initial fluctuation of the Higgs field; thus the Higgs field is correlated throughout the observed Universe. Some monopoles will still be produced by thermal fluctuations, but for 10^{16} GeV monopoles this number is acceptably small.

The horizon problem disappears for essentially this same reason. The observed universe evolves from a single causally connected FR in the De Sitter phase. The exponential expansion of the single fluctuation causes a very small homogeneous region to grow large enough to encompass the observed Universe.

Finally, the flatness problem is avoided by the dynamics of the exponential expansion of the

FR. As ϕ begins to move very slowly down the potential, the gravitational dynamics is governed by the energy density ρ_0. Assuming that the FR (or a small piece of it) can be approximated locally by a Robertson–Walker metric, then the scale factor evolves according to the standard equation

$$(\dot{R}/R)^2 = \tfrac{8}{3}\pi G\rho - k/R^2. \qquad (19)$$

In this language, the flatness problem is the problem of understanding why the second term on the right-hand side is so extraordinarily small. But as the FR expands exponentially, the energy density ρ remains very nearly constant at ρ_0, while the k/R^2 term falls off as the square of the exponential factor. Even if the k/R^2 term is initially very large, it is negligible by the time the exponential expansion ends.

Except maybe for a very narrow range of parameters, this suppression of the curvature term will almost certainly vastly exceed that required by present observations. This leads to the prediction that the value of Ω today should be equal to unity to within an extraordinary degree of accuracy. At present, this is the only prediction of the inflationary scenario.

In conclusion, at present I do not really know if the details of the Linde–Albrecht–Steinhardt ending to the inflationary scenario will work out or not. If they do, we would have an elegant solution to some very important problems in cosmology: the monopole problem, the horizon problem and the flatness problem. Such a scenario would also explain the origin of all the matter, energy and entropy in the Universe.

Possibly the most far-reaching recent development in the study of cosmology is the realization that our Universe may contain no conserved quantities whatever. If this is true, then it is very tempting to assume that the Universe emerged from nothing or from almost nothing. One scenario of this type is the inflationary Universe.

It is often said that there is no such thing as a free lunch. It now appears possible, however, that the Universe is a free lunch.

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Discussion

S. P. Bhavsar (*Astronomy Centre, University of Sussex, U.K.*). Is it not possible that our present Universe is in a secondary false vacuum state, having decayed to it from some previous false vacuum? We are then not constrained to a value of $\Omega = 1$ to a high degree of accuracy for the present Universe, but can have a value of Ω arbitrarily close to 1 (say 0.1) by choosing the appropriate secondary state.

A. H. Guth. Perhaps I do not understand the question, but I do not see that the expected present value of Ω depends in any way on whether the present state of the Universe is metastable (i.e. near a false vacuum) or stable (i.e. near the true vacuum). If there was a previous era of exponential inflation, then the value of Ω depends sensitively on the time period Δt that this era lasted: the value of $\Omega - 1$ is suppressed during this era by a factor of $e^{2\chi\Delta t}$. If $\Omega - 1$ were of order unity before the inflation, it would be of order unity today if $\chi\Delta t \approx 56$. If, however, Δt were 10% larger than this, the present value of $\Omega - 1$ would be of order $e^{-11.2} \approx 10^{-5}$. Thus the inflationary scenario does not preclude the possibility that $\Omega \approx 0.1$, but the overwhelming likelihood would be $\Omega = 1$ to a high degree of accuracy.